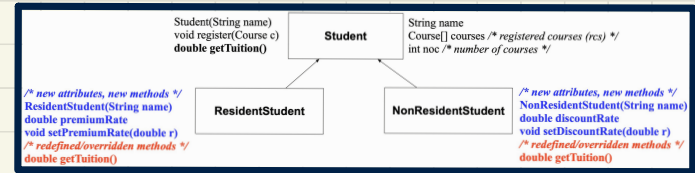


Casting Arguments

```
void addRS(ResidentStudent rs)
```



sms.addRS((**ResidentStudent**) s) compiles?

```
1 Student s = new Student("Stella");
2 /* s' ST: Student; s' DT: Student */
3 StudentManagementSystem sms = new StudentManagementSystem();
4 sms.addRS(s); ✗
```

ClassCastException?

```
1 Student s = new NonResidentStudent("Nancy");
2 /* s' ST: Student; s' DT: NonResidentStudent */
3 StudentManagementSystem sms = new StudentManagementSystem();
4 sms.addRS(s); ✗
```

ClassCastException?

```
1 Student s = new ResidentStudent("Rachael");
2 /* s' ST: Student; s' DT: ResidentStudent */
3 StudentManagementSystem sms = new StudentManagementSystem();
4 sms.addRS(s); ✗
```

ClassCastException?

sms.addRS((**ResidentStudent**) nrs) compiles?

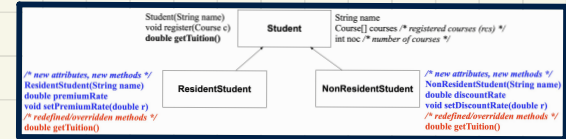
```
1 NonResidentStudent nrs = new NonResidentStudent();
2 /* ST: NonResidentStudent; DT: NonResidentStudent */
3 StudentManagementSystem sms = new StudentManagementSystem();
4 sms.addRS(nrs); ✗
```

A Polymorphic Collection of Students

```

1 ResidentStudent rs = new ResidentStudent("Rachael");
2 rs.setPremiumRate(1.5);
3 NonResidentStudent nrs = new NonResidentStudent("Nancy");
4 nrs.setDiscountRate(0.5);
5 StudentManagementSystem sms = new StudentManagementSystem();
6 sms.addStudent(rs); /* polymorphism */
7 sms.addStudent(nrs); /* polymorphism */
8 Course eeecs2030 = new Course("EECS2030", 500.0);
9 sms.registerAll(eeecs2030);
10 for(int i = 0; i < sms.numberOfStudents; i++) {
11     /* Dynamic Binding:
12      * Right version of getTuition will be called */
13     System.out.println(sms.students[i].getTuition());
14 }

```



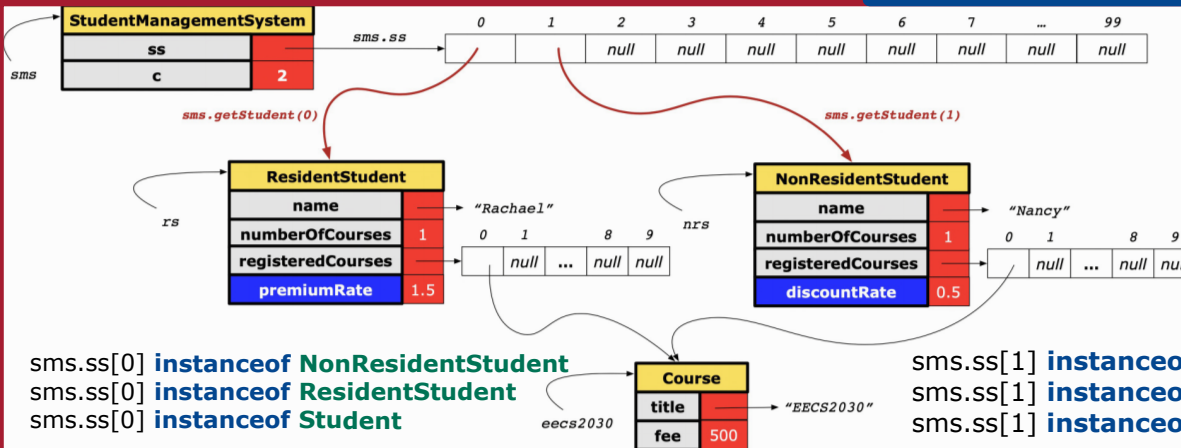
```

class StudentManagementSystem {
    Student[] students;
    int numofStudents;

    void addStudent(Student s) {
        students[numofStudents] = s;
        numofStudents++;
    }

    void registerAll(Course c) {
        for(int i = 0; i < numofStudents; i++) {
            students[i].register(c)
        }
    }
}

```



Polymorphic Return Types

```

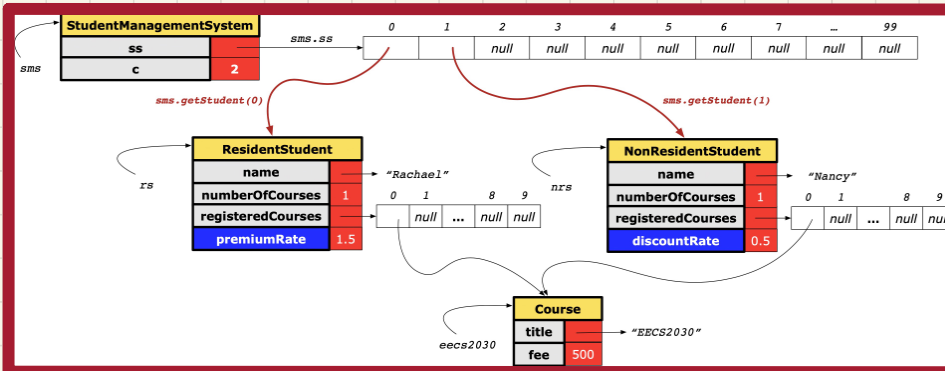
Course eecs2030 = new Course("EECS2030", 500);
ResidentStudent rs = new ResidentStudent("Rachael");
rs.setPremiumRate(1.5); rs.register(eecs2030);
NonResidentStudent nrs = new NonResidentStudent("Nancy");
nrs.setDiscountRate(0.5); nrs.register(eecs2030);
StudentManagementSystem sms = new StudentManagementSystem();
sms.addStudent(rs); sms.addStudent(nrs);
Student s = sms.getStudent(0); /* dynamic type of s? */

    static return type: Student
print(s instanceof Student && s instanceof ResidentStudent); /* true */
print(s instanceof NonResidentStudent); /* false */
print(s.getTuition()); /* Version in ResidentStudent called: 750 */
ResidentStudent rs2 = sms.getStudent(0); x
s = sms.getStudent(1); /* dynamic type of s? */

    static return type: Student
print(s instanceof Student && s instanceof NonResidentStudent); /* true */
print(s instanceof ResidentStudent); /* false */
print(s.getTuition()); /* Version in NonResidentStudent called: 250 */
NonResidentStudent nrs2 = sms.getStudent(1); x
    
```

```

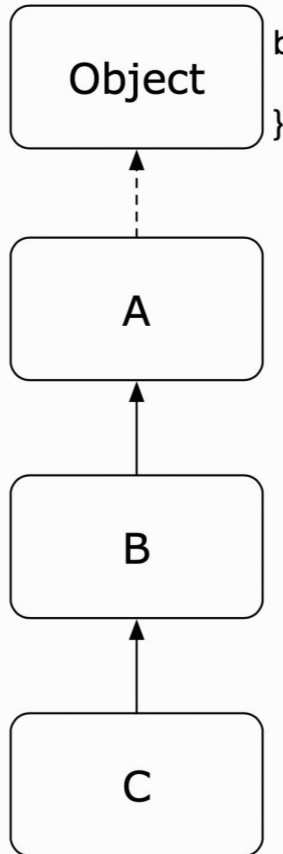
class StudentManagementSystem {
    Student[] ss; int c;
    void addStudent(Student s) { ss[c] = s; c++; }
    Student getStudent(int i) {
        Student s = null;
        if(i < 0 || i >= c) {
            throw new IllegalArgumentException("Invalid index");
        }
        else {
            s = ss[i];
        }
        return s;
    }
}
    
```



Summary: Type Checking Rules

CODE	CONDITION TO BE TYPE CORRECT
<code>x = y</code>	Is <code>y</code> 's ST a descendant of <code>x</code> 's ST ?
<code>x.m(y)</code>	Is method <code>m</code> defined in <code>x</code> 's ST ? Is <code>y</code> 's ST a descendant of <code>m</code> 's parameter's ST ?
<code>z = x.m(y)</code>	Is method <code>m</code> defined in <code>x</code> 's ST ? Is <code>y</code> 's ST a descendant of <code>m</code> 's parameter's ST ? Is ST of <code>m</code> 's return value a descendant of <code>z</code> 's ST ?
<code>(C) y</code>	Is <code>C</code> an ancestor or a descendant of <code>y</code> 's ST ?
<code>x = (C) y</code>	Is <code>C</code> an ancestor or a descendant of <code>y</code> 's ST ? Is <code>C</code> a descendant of <code>x</code> 's ST ?
<code>x.m((C) y)</code>	Is <code>C</code> an ancestor or a descendant of <code>y</code> 's ST ? Is method <code>m</code> defined in <code>x</code> 's ST ? Is <code>C</code> a descendant of <code>m</code> 's parameter's ST ?

Overridden Methods and Dynamic Binding (1)



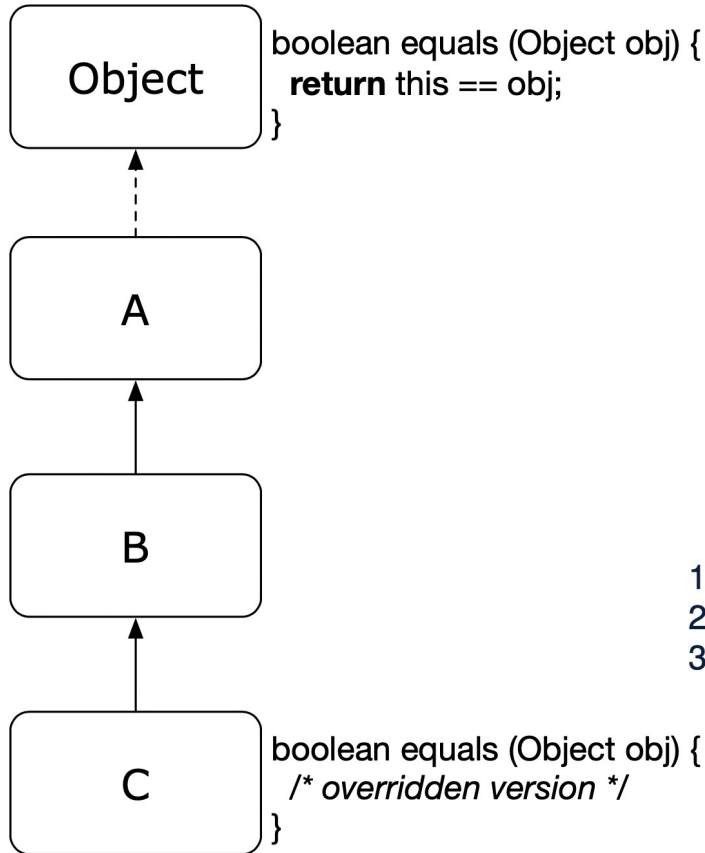
```
boolean equals (Object obj) {  
    return this == obj;  
}
```

```
class A {  
    /*equals not overridden*/  
}  
class B extends A {  
    /*equals not overridden*/  
}  
class C extends B {  
    /*equals not overridden*/  
}
```

```
1 Object c1 = new C();  
2 Object c2 = new C();  
3 println(c1.equals(c2));
```

L3 calls which version of equals?

Overridden Methods and Dynamic Binding (2)

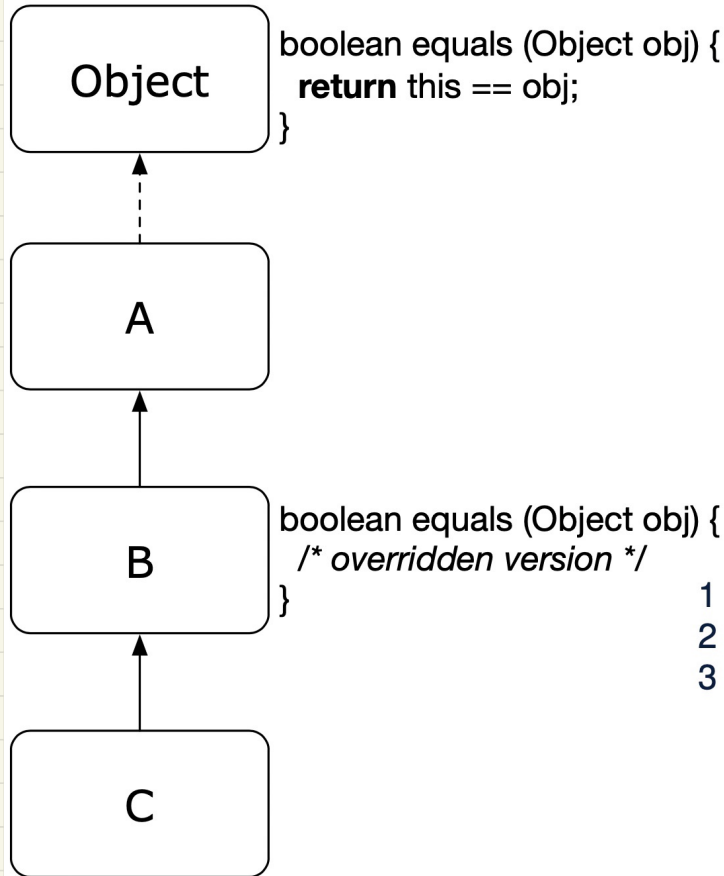


```
class A {  
    /*equals not overridden*/  
}  
class B extends A {  
    /*equals not overridden*/  
}  
class C extends B {  
    boolean equals (Object obj) {  
        /* overridden version */  
    }  
}
```

```
1 Object c1 = new C();  
2 Object c2 = new C();  
3 println(c1.equals(c2));
```

L3 calls which version of equals?

Overridden Methods and Dynamic Binding (3)



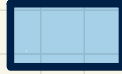
```
class A {  
    /*equals not overridden*/  
}  
class B extends A {  
    boolean equals (Object obj) {  
        /* overridden version */  
    }  
}  
class C extends B {  
    /*equals not overridden*/  
}
```

```
1 Object c1 = new C();  
2 Object c2 = new C();  
3 println(c1.equals(c2));
```

L3 calls which version of equals?

Solving a Problem **Recursively**

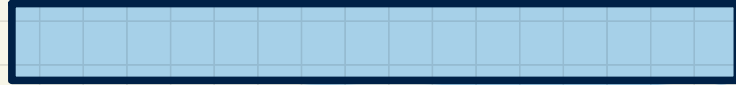
Given a **small** problem:



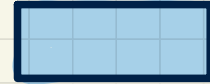
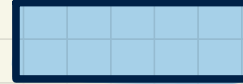
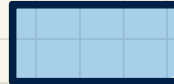
Solve it **directly**:



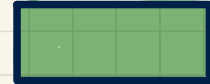
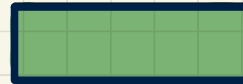
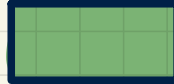
Given a **big** problem:



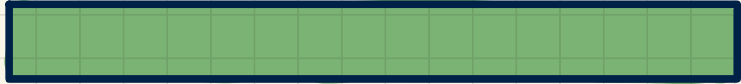
Divide it into **smaller** problems:



Assume solutions to **smaller** problems:



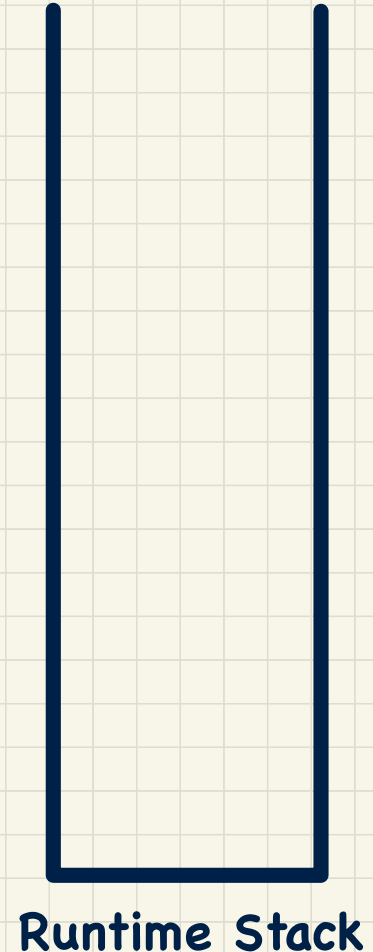
Combine solutions to **smaller** problems:



```
m(i) {  
  if(i == ...) { /* base case: do something directly */ }  
  else {  
    m(j); /* recursive call with strictly smaller value */  
  }  
}
```

Tracing Recursion via a Stack

- When a method is called, it is **activated** (and becomes **active**) and **pushed** onto the stack.
- When the body of a method makes a (helper) method call, that (helper) method is **activated** (and becomes **active**) and **pushed** onto the stack.
 - ⇒ The stack contains activation records of all **active** methods.
 - **Top** of stack denotes the current point of execution.
 - Remaining parts of stack are (temporarily) **suspended**.
- When entire body of a method is executed, stack is **popped**.
 - ⇒ The current point of execution is returned to the new **top** of stack (which was **suspended** and just became **active**).
- Execution terminates when the stack becomes **empty**.



Recursive Solution: factorial

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1 & \text{if } n \geq 1 \end{cases}$$

Recursive Solution in Java: factorial

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{if } n \geq 1 \end{cases}$$

```
int factorial(int n) {  
    int result;  
    if(n == 0) { /* base case */ result = 1; }  
    else { /* recursive case */  
        result = n * factorial(n - 1);  
    }  
    return result;  
}
```

Example: factorial(3)

Runtime Stack

Common Errors of Recursion (1)

```
int factorial (int n) {  
    return n * factorial (n - 1);  
}
```

Common Errors of Recursion (2)

```
int factorial(int n) {  
    if(n == 0) { /* base case */ return 1; }  
    else { /* recursive case */ return n * factorial(n); }  
}
```